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International Journal of Control, Automation and Systems 18(1) (2020) 217-224

ISSN:1598-6446 (print version)
eISSN:2005-4092 (electronic version)

To link to this article:
http://dx.doi.org/10.1007/s12555-018-0749-7

Springer

International Journal of Control, Automation and Systems 18(1) (2020) 217-224
http://dx.doi.org/10.1007/s12555-018-0749-7
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Active Balancing Control for Unmanned Bicycle Using Scissored-pair Control Moment Gyroscope

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Abstract: A control moment gyroscope (CMG) is capable of generating strong restoration torque with a relatively small-sized flywheel by changing the direction of flywheel momentum. Because the CMG is energy efficient, it has been used for active balancing of a bicycle that has an unstable equilibrium point in its dynamics. A single CMG generates not only the restoration torque component but also an additional unwanted torque component. In contrast, a scissored-pair CMG cancels out the unwanted torque and doubles the restoration torque. This study deals with active balancing to implement an unmanned bicycle by using a scissored-pair CMG. Based on the inverted pendulum dynamics model of a bicycle, a linear quadratic regulation algorithm is presented for balancing control. In order to investigate active balancing, a miniaturized bicycle system is developed, and its 3D solid model is created to obtain the parameter values of the dynamics model. Two kinds of disturbances exist that can cause the instability of a bicycle with an unstable equilibrium point: impulsive external disturbance and static constant disturbance. Experimental results show the performance of active balancing for a bicycle with a scissored-pair CMG against those disturbances.

Keywords: Active balancing, disturbance, inverted pendulum, linear quadratic regulation, scissored-pair CMG, unmanned bicycle.

1. INTRODUCTION

Bicycles are a familiar transportation vehicle, having been in existence since the early 19th century. The bicycle is a typical dynamic system with unstable equilibrium points similar to an inverted pendulum, unicycle robot, or ballbot. The unmanned bicycle has drawn increasing attention as an educational and experimental test bed for automatic control theory [1, 2]. From a practical point of view, the bicycle has good mobility in narrow and cluttered environments and has high energy efficiency because of low friction with the ground. There have thus been many research results on dynamics modeling and control methods for the unmanned bicycle [3–7].

Self-balancing is an essential feature for the implementation of unmanned bicycles. There are two methods for self-balancing control: 1) by using only the steering and the speed of a bicycle itself; and 2) by using additional active balancing actuators. Tanaka *et al.* presented a linear approximation of the dynamic equation including the centrifugal force for a bicycle, and they proposed a PID control algorithm for the input variables of steering angle and driving speed to achieve self-balancing [8]. Park *et al.* obtained the steering angle and driving speed by using their sliding patch and stuck control algorithm [9], and Guo *et al.* proposed a feedback linearization control method [10] for the self-balancing of a bicycle. Defoort *et al.* studied a disturbance observer and sliding controller for the steering angle and driving speed, respectively of an unmanned bicycle to attain self-balancing [11]. These control algorithms compensate the falling-down force with the centrifugal force generated by the driving speed and steering angle of the bicycle. The falling-down force is caused by gravity and the centrifugal force is generated by the traveling speed together with the turning radius controlled by the steering angle. These methods do not reflect the active balancing motion of a human rider and are vulnerable to external disturbances.

In order to overcome these problems, active balancing methods adopt additional actuators for the self-balancing of the bicycle. In [12], Lee *et al.* adopted a mass balancer as the actuator to implement active balancing. The mass balancer has linear motion in the lateral direction of a bicycle to mimic a human rider. Iuchi *et al.* used an inverted pendulum type balancing weight for active balancing [13]. They treated the bicycle plus balancing weight as a double pendulum problem. Hwang *et al.* presented a variable structure control algorithm for balanc-

Manuscript received October 17, 2018; revised May 5, 2019 and July 24, 2019; accepted August 14, 2019. Recommended by Associate Editor Chang Kyung Ruoo under the direction of Editor Won-jong Kim. This research was supported by the research program funded by National Research Foundation (Ministry of Education) of Korea (NRF-2018R1D1A1B07044841).

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ing using a similar pendulum balancer [14]. In [15], Keo *et al.* showed that simultaneous control of the balancer and steering handlebar is advantageous for self-balancing of a bicycle. A unicycle robot was a platform for self-balancing with a mass balancer in [16], and a flywheel balancer with changeable configuration was presented in [17]. Active balancing methods using an additional actuator have robustness against external disturbances because they are independent of the driving speed and the steering angle of the bicycle.

Beside a balancing weight, the reaction wheel and control moment gyroscope (CMG) are commonly used as the actuators for active balancing. The reaction wheel is a momentum wheel spinning by a rotor, and it generates torque by changing the spinning direction and speed of the wheel. On the other hand, the CMG generates torque by changing the direction of the spinning axis of a flywheel rotating at constant speed [18, 19]. A motorized gimbal is used to change the direction of the spinning axis [20]. The "Murata boy" robot developed by Murata is an example of an active balancing bicycle using a reaction wheel [21]. Kim *et al.* used a similar reaction wheel for balancing of a small bicycle robot in [22].

A reaction wheel is simple in its structure and easy to develop; however, it is weak in torque generation and demands a relatively large flywheel disk. In contrast, a CMG is capable of torque generation with a relatively smaller flywheel and is generally higher in energy efficiency than a reaction wheel [23]. The CMG has been used in satellite attitude control system [24, 25] and adopted for active balancing of a bicycle in recent years [26–29].

A single CMG generates a torque component in an unwanted direction. Because it is impossible to compensate for the unwanted torque solely by gimbal control, the unwanted torque can disturb the stability of a bicycle in motion [30]. However, it is possible to overcome this problem by using a scissored-pair CMG consisting of two CMGs spinning in counter directions to cancel the unwanted torques out [31]. Brown *et al.* proposed to use the scissored-pair CMG as space-robotics actuators by taking advantage of the reactionless actuation property of the CMG. The energy required to actuate a robotic linkage by CMGs was compared with that by direct drive motors [32]. Chase adopted the scissored-pair CMG for the driving system of spherical robots to solve the power constraints in the common barycenter-offset designs of spherical robots [33].

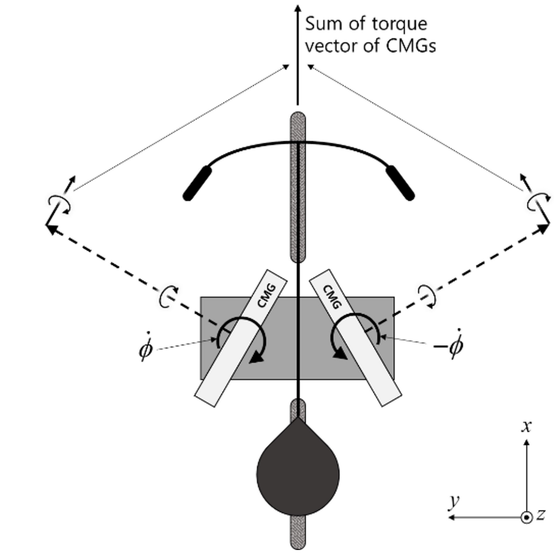
This study presents an active balancing control for a bicycle with unstable equilibrium points using a scissored-pair CMG. A linear quadratic regulation (LQR) algorithm is proposed for the gimbal motor control of the scissored-pair CMG in order to realize active balancing. Some existing control algorithms regard the gimbal rate of the CMGs as a control input variable, not a state variable; these do not guarantee the convergence of the gimbal angle in the steady-state [29]. The LQR algorithm in this study takes both of the tilt angle of the bicycle and the gimbal rate as state variables to guarantee the stability of the equilibrium point of the bicycle with a scissored-pair CMG. In order to investigate the proposed active balancing, a miniaturized bicycle system with a scissored-pair CMG was constructed. A 3D solid model of the bicycle system was also created to extract the parameter values of the dynamics equation. Impulsive disturbance as well as static constant disturbance cause instability of a bicycle with an unstable equilibrium point. Experiments were performed to verify the robustness of the proposed active balancing of the unmanned bicycle against those disturbances.

2. UNMANNED BICYCLE WITH SCISSORED-PAIR CMG

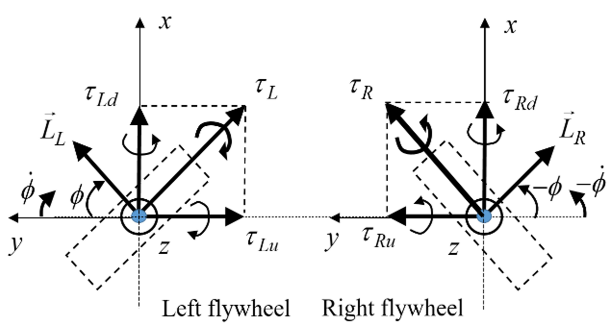
Fig. 1 shows an application of the scissored-pair CMG for active balancing of a bicycle. The scissored-pair CMG consists of two flywheels and gimbals as shown in Fig. 1(a); the flywheels are spinning in counter directions to each other at a constant speed and the gimbals are mechanically linked to rotate in opposed directions. An angular momentum vector of a spinning flywheel is represented as $\vec{L}$ with its magnitude as $L_f = I_f\omega_f$, where I_f and ω_f are the inertia and the angular velocity of the spinning wheel, respectively. The angular momentum vectors of the left and the right flywheels are denoted as $\vec{L}_L$ and $\vec{L}_R$ in Fig. 1. The generated torque from a CMG according to gimbal rate is represented as

$$\tau = L_f\dot{\phi}\cos\phi\vec{i} + L_f\dot{\phi}\sin\phi\vec{j}. \tag{1}$$

The first torque component, $\tau_d = L_f\dot{\phi}\cos\phi\vec{i}$ in (1), is a restoration torque against the rolling about x-axis, i.e., the falling-down of the bicycle. The remaining $\tau_u = L_f\dot{\phi}\sin\phi\vec{j}$ in (1) is an unwanted torque component that may affect the balance of a bicycle as disturbance when a single CMG is used for a bicycle. However, when a scissored-pair CMG is used as shown in Fig. 1(b), the unwanted torque components, τ_{Lu} and τ_{Ru} from the two CMGs on either side have counter directions and cancel each other out. Because τ_{Ld} and τ_{Rd} have the same rotational directions about the x-axis, the restoration forces against the falling-down of a bicycle are increased. At $\phi = 0°$, the restoration force of the scissored-pair CMG is double that of a single CMG. Thus, the gimbal angle should be controlled for $\vec{L}$ to stay around the equilibrium, $\phi \approx 0°$ by the gimbal motor. The scissored-pair CMG is advantageous for the balancing control of a bicycle because of the symmetric structure as well as the increased restoration force.



(a) Bicycle with scissored-pair CMG.



(b) Vector diagram.

Fig. 1. Scissored-pair CMG.

3. ACTIVE BALANCING CONTROL

Dynamics control for self-balancing of a bicycle requires the dynamics equation with parameter values. Table 1 summarizes the parameters. This study takes into consideration not only the dynamics of a bicycle with the CMGs, but also the dynamics of a gimbal motor for balancing control of the entire bicycle system.

Under the assumption that the longitudinal speed of a bicycle is slow enough, the dynamics of a bicycle in the lateral direction is modeled as that of an inverted pendulum. Equation (2) represents the inverted pendulum dynamics; the last term on the right-hand side denotes the torque generated by two CMG flywheels. The dynamics of the scissored-pair CMG consisting of two flywheels is described by (3). Two gimbals are driven by one gimbal motor. The dynamics of the gimbal motor is described by (4).

$$J_b\ddot{\theta} = mgl\sin\theta - 2L_f\dot{\phi}\cos\phi, \tag{2}$$

$$2J_C\ddot{\phi} = 2L_f\dot{\theta}\cos\phi + \tau_g, \tag{3}$$

Table 1. Parameters.

Symbol	Description
θ	Tilt angle of bicycle
ϕ	Angle of CMG gimbal
J_b	Moment of inertia of bicycle about x axis
J_C	Moment of inertia of CMG flywheel about z axis
m	Mass of bicycle
l	Distance from ground to enter of bicycle mass
g	Gravitational acceleration
L_f	Angular momentum of a flywheel
τ_g	Torque from gimbal motor
V	Control input to gimbal motor (Voltage)
k_t	Torque constant of gimbal motor
k_b	Back electromotive force constant
R	Resistance of gimbal motor coil

$$\tau_g = \frac{k_t}{R}V - \frac{k_t k_b}{R}\dot{\phi}. \tag{4}$$

In [29], Yetkin *et al.* designed a nonlinear variable structure control for active balancing of a bicycle with a single CMG. The sliding surface in [29] contains the state about the tilt angle only; it does not guarantee the stability of every state by including the gimbal angle in the dynamics model. This study presents an LQR control that takes both of the tilt angle and the gimbal rate as state variables in order to guarantee the stability of the equilibrium state. Equations (2) and (3) are linearized into (5) and (6) by approximation around the equilibrium point.

$$J_b\ddot{\theta} = mgl\theta - 2L_f\dot{\phi}, \tag{5}$$

$$2J_C\ddot{\phi} = 2L_f\dot{\theta} + \tau_g. \tag{6}$$

The system of (4), (5), and (6) is represented in matrix form as (7).

$$\begin{gathered}\dot{X} = AX + Bu,\\ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{mgl}{J_b} & 0 & 0 & -\frac{2L_f}{J_b} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{L_f}{J_C} & 0 & -\frac{k_t k_b}{2RJ_C} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{2RJ_C} \end{bmatrix},\end{gathered} \tag{7}$$

where the state variable, X and the control input, u are as follows:

$$X = \begin{bmatrix} \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^t, \quad u = V. \tag{8}$$

The cost function for the LQR control is defined as

$$J = \int_0^\infty \left(X^t QX + Ru^2\right) dt, \tag{9}$$

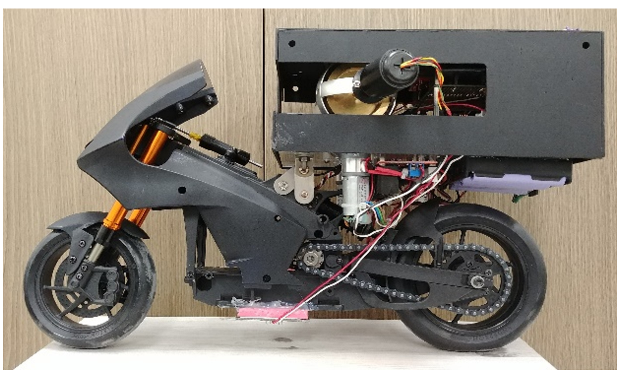
where Q and R are the weights on the state variable and the input variable, respectively. The solution of the input variable to minimize the cost function is given as

$$u = -KX, \tag{10}$$

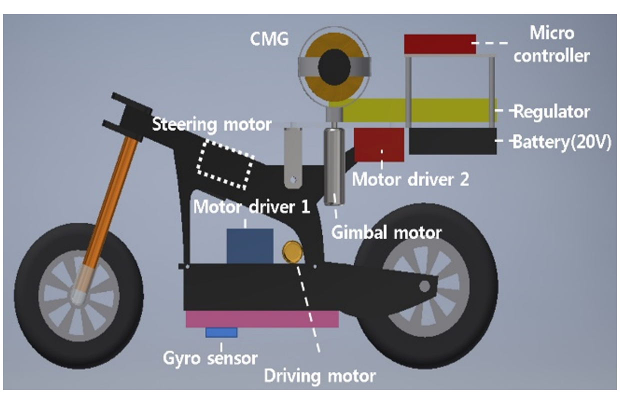
where K is the state feedback gain [34].

4. EXPERIMENTAL RESULTS

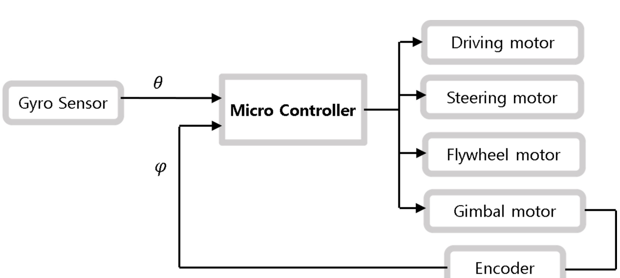
In order to implement the active balancing control presented in this study, a test platform was constructed using an actual miniaturized bicycle and experiments were carried out. Fig. 2 shows the experimental platform of the bicycle system presented in this study. The bicycle has control subsystem consisting of sensors, actuators, and a microcontroller (STM32F407, 84MHz). The sensors include a motion tracking device (MPU-6050) to measure the tilt angle of the bicycle and optical encoders to measure the gimbal angle of each CMG gimbal motor. The bicycle has a driving motor and a steering motor besides a flywheel motor and a gimbal motor in each of two CMGs.



(a) Actual miniaturized bicycle.



(b) Parts layout.



(c) Miniaturized bicycle with scissored-pair CMG.

Fig. 2. Miniaturized bicycle with scissored-pair CMG.

Two CMGs were deployed at both side of the bicycle. The commercially available CMG [35] was adopted and its gimbal motor was replaced for precise feedback control in this study. Because there was no mechanical linkage between the two CMGs, the rotation angles ϕ and the angular velocities $\dot{\phi}$ of two gimbal motors were controlled to keep the same values, in order to implement the scissored-pair CMG. The control algorithm is running in the microcontroller and the microcontroller sends the measurement data to a remote monitoring PC through Bluetooth wireless channel.

The flywheel speed of the CMG was constant as around 11,000 revolutions per minute. Because the flywheel rotating with high speed causes mechanical vibration in the gyro sensor, a soft rubber was attached with the sensor module in order to reduce the vibration delivered to MPU-6050 sensor. The well-known complementary filter algorithm that combines the gyro sensor data and the accelerometer data from MPU-6050 with adjustable weights was applied for removing the noise component in the measured tilt angle.

4.1. Solid model and parameters of dynamics equation

A 3D solid model of the bicycle was created to extract the parameter values of the dynamics model. Fig. 3 shows the solid model in a CAD system [36, 37]. The parameter values of the gimbal motor were obtained from the specifications of the motor. Table 2 summarizes the parameter values.

The weighting matrices for the cost function, (9), of the LQR control are defined as (11).

$$Q = \begin{bmatrix} 89.0 & 0 & 0 & 0 \\ 0 & 15.0 & 0 & 0 \\ 0 & 0 & 26.0 & 0 \\ 0 & 0 & 0 & 4.7 \end{bmatrix}, \quad R = 1.0. \tag{11}$$

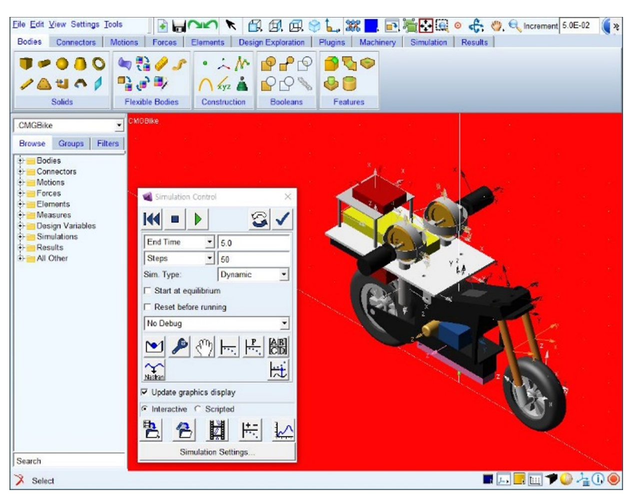


Fig. 3. Solid model of unmanned bicycle.

Table 2. Parameter values.

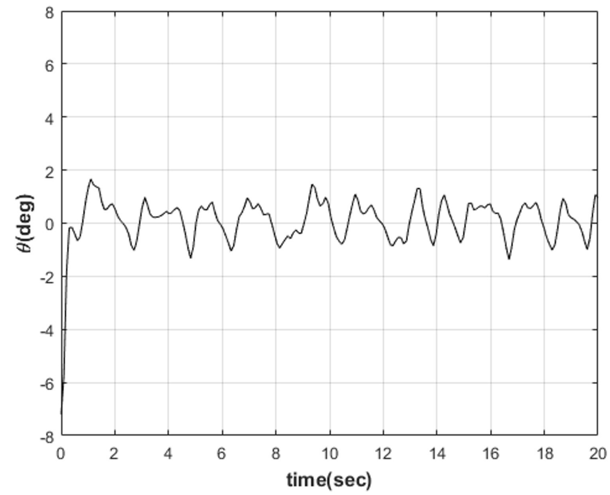
Parameter	Value
J_b	6.52×10^{-2} kg·m^2
J_C	2.29×10^{-4} kg·m^2
m	2.36 kg
g	9.81 m/s^2
l	1.48×10^{-1} m
L_f	5.83×10^{-2} kg·m$^2/s$
k_t	2.72×10^{-1} kg·m$^2/(s^2 \cdot A)$
k_b	2.72×10^{-1} V/(rad/s)
R	12.0 Ω

The biggest weighting value was assigned to the tilt angle, θ, that is directly affected by the gravitational force. The angular velocity of the gimbal motor, $\dot{\phi}$, had the smallest weighting value. The LQR control problem was formulated by the dynamics equation using the parameter values in Table 2 and the cost function (9) with the weighting factors in (11). The solution of the problem was obtained by using the MATLAB LQR tool as follows:

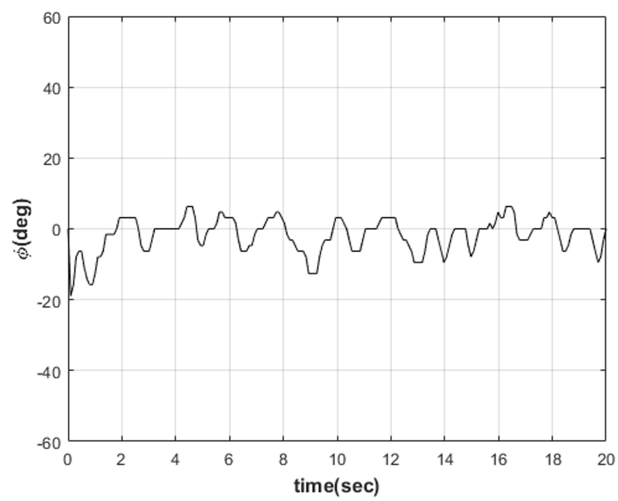
$$K = \begin{bmatrix} -180.6 & -20.6 & -5.1 & 2.2 \end{bmatrix}. \tag{12}$$

4.2. Self-uprising from an initial tilt angle

From the initial state with a particular tilt angle, the bicycle was capable of uprising to an equilibrium state and keeping its balance using the scissored-pair CMG. The result of the experiment is presented in Fig. 4 where the initial tilt angle was around $\theta \approx -8°$. In the experimental result, the tilt angle stayed within $|\theta| < 1.0°$ as time goes; this implies that the bicycle was maintaining balance in the steady-state. A video clip of the experiments in this study is found in [38].



(a) Tilt angle θ of unmanned bicycle.



(b) Gimbal angle ϕ of CMG.

Fig. 4. Experimental result for self-uprising.

4.3. Robustness against external impulsive disturbance

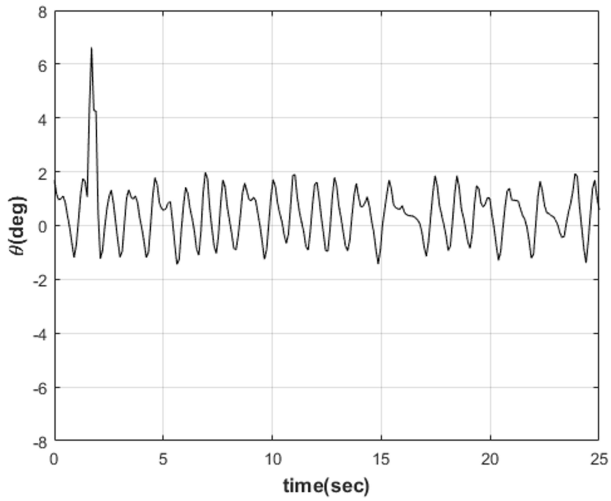
Fig. 5 shows the experimental result to verify the robustness of active balancing against external disturbance. An impulsive disturbance is applied to the bicycle at around $t = 2$ sec. As before, the bicycle with the active balancing control maintained its balance against the impulsive disturbances.

4.4. Robustness against static disturbance in circle driving

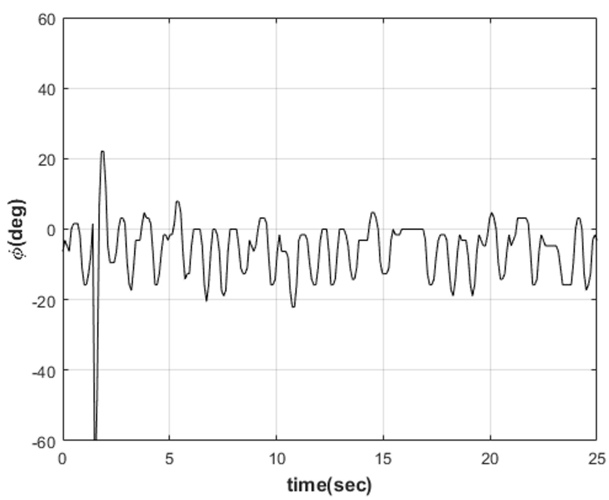
When a bicycle with a single CMG moves in a circular trajectory, there is a rotational motion about the z-axis that changes the direction of the angular momentum of the flywheel. It is noted that the direction change generates additional torque about the longitudinal axis of the bicycle. At equilibrium, the additional torque can be a static disturbance which hinders the bicycle's balance. In the case of the scissored-pair CMG, those additional torques generated by each of the CMGs are canceled out and the bicycle is able to keep its balance. Fig. 6 shows the experimental results for the bicycle with a scissored-pair CMG moving on a circular trajectory. The period of the circular trajectory was $T = 6$ sec. and the additional static disturbance of $\tau = 2L_f \cdot \frac{2\pi}{T} \approx 0.122$ Nm was generated continuously by the rotational motion in the circle driving if the bicycle had a single CMG. Despite the additional disturbance, the tilt angle of the bicycle with the scissored-pair CMG stayed around $0° < \theta < 2°$ in the steady-state as shown in Fig. 6(a) [39].

5. CONCLUSION

A bicycle is a dynamic system with unstable equilibrium points, and self-balancing is essential to implement an unmanned bicycle. This study presented an active balancing control for an unmanned bicycle using a scissored-

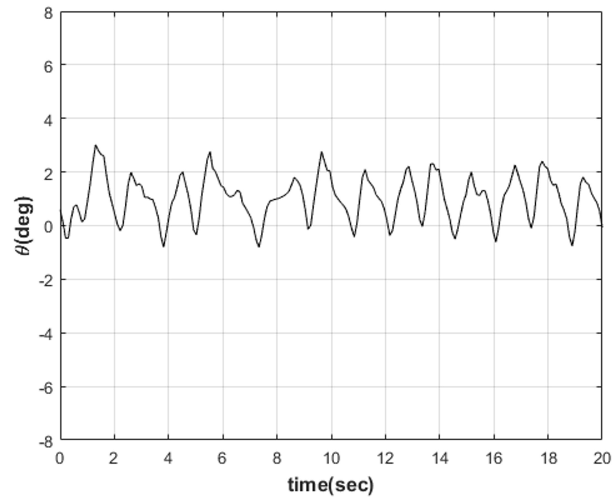


(a) Tilt angle θ of unmanned bicycle.

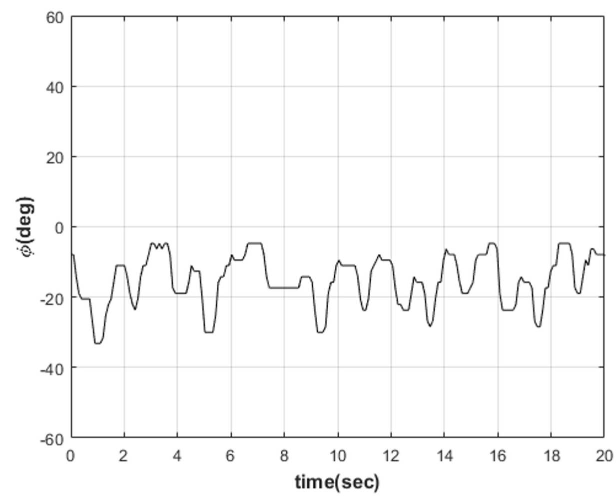


(b) Gimbal angle ϕ of CMG.

Fig. 5. Experimental result for external disturbance.



(a) Tilt angle θ of unmanned bicycle.



(b) CMG gimbal angle ϕ.

Fig. 6. Experimental result for circle driving.

pair CMG. A single CMG generates an unwanted torque component as well as the restoration torque. By contrast, the scissored-pair CMG cancels out the unwanted torque and doubles the restoration torque. Based on an inverted pendulum dynamics model including the CMG and the gimbal motor dynamics, an LQR algorithm was proposed for the active balancing control of a bicycle with the scissored-pair CMG in this study. A miniaturized bicycle system was implemented to investigate the active balancing control. A 3D solid model of the bicycle system was created to extract the parameter values of the dynamics equation. Two independent CMG modules were used in this study to realize the scissored-pair CMG by controlling their gimbal motors to keep the same rotation angles. By taking both of the tilt angle of the bicycle and the gimbal rate as state variables, an LQR control algorithm is presented for the balancing control. Although there were some mismatches in the two CMGs, experimental results show the performance of active balancing for a bicycle with an unstable equilibrium point and robustness against external impulsive disturbance and additional static disturbance caused by circle driving.

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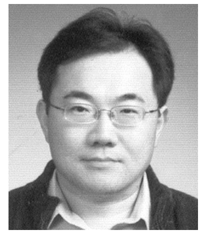

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